

**Exercise 34**

(a) Use a graph of

$$f(x) = \frac{\sqrt{3+x} - \sqrt{3}}{x}$$

to estimate the value of  $\lim_{x \rightarrow 0} f(x)$  to two decimal places.

(b) Use a table of values of  $f(x)$  to estimate the limit to four decimal places.

(c) Use the Limit Laws to find the exact value of the limit.

**Solution**

Evaluate the given function at several values of  $x$  near 0.

$$\left. \frac{\sqrt{3+x} - \sqrt{3}}{x} \right|_{x=-0.1} = 0.291122$$

$$\left. \frac{\sqrt{3+x} - \sqrt{3}}{x} \right|_{x=-0.01} = 0.288916$$

$$\left. \frac{\sqrt{3+x} - \sqrt{3}}{x} \right|_{x=-0.001} = 0.288699$$

$$\left. \frac{\sqrt{3+x} - \sqrt{3}}{x} \right|_{x=-0.0001} = 0.288678$$

$$\left. \frac{\sqrt{3+x} - \sqrt{3}}{x} \right|_{x=0.0001} = 0.288673$$

$$\left. \frac{\sqrt{3+x} - \sqrt{3}}{x} \right|_{x=0.001} = 0.288651$$

$$\left. \frac{\sqrt{3+x} - \sqrt{3}}{x} \right|_{x=0.01} = 0.288435$$

$$\left. \frac{\sqrt{3+x} - \sqrt{3}}{x} \right|_{x=0.1} = 0.286309$$

Therefore,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} &= \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} \cdot \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}} = \lim_{x \rightarrow 0} \frac{(3+x) - 3}{x(\sqrt{3+x} + \sqrt{3})} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{3+x} + \sqrt{3}} \\ &= \frac{1}{2\sqrt{3}} \approx 0.288675. \end{aligned}$$

The graph of  $y = f(x)$  versus  $x$  confirms the result.

