Exercise 36

Use the Squeeze Theorem to show that

$$\lim_{x\to 0} \sqrt{x^3 + x^2}\sin\frac{\pi}{x} = 0$$

Illustrate by graphing the functions f, g, and h (in the notation of the Squeeze Theorem) on the same screen.

Solution

Since

$$-\sqrt{x^3 + x^2} \le \sqrt{x^3 + x^2}\sin\frac{\pi}{x} \le \sqrt{x^3 + x^2}$$

and

$$\lim_{x\to 0}\left(-\sqrt{x^3 + x^2}\right) = \lim_{x\to 0}\left(\sqrt{x^3 + x^2}\right) = 0,$$

then

$$\lim_{x\to 0} \sqrt{x^3 + x^2}\sin\frac{\pi}{x} = 0$$

by the squeeze theorem. The graph below of each of the functions versus x confirms this result.