

Exercise 44

Find the limit, if it exists. If the limit does not exist, explain why.

$$\lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x}$$

Solution

Evaluate the left-hand and right-hand limits separately.

$$\lim_{x \rightarrow -2^-} \frac{2 - |x|}{2 + x} = \lim_{x \rightarrow -2^-} \frac{2 - (-x)}{2 + x} = \lim_{x \rightarrow -2^-} \frac{2 + x}{2 + x} = \lim_{x \rightarrow -2^-} (1) = 1$$

$$\lim_{x \rightarrow -2^+} \frac{2 - |x|}{2 + x} = \lim_{x \rightarrow -2^+} \frac{2 - (-x)}{2 + x} = \lim_{x \rightarrow -2^+} \frac{2 + x}{2 + x} = \lim_{x \rightarrow -2^+} (1) = 1$$

Since both these limits are equal,

$$\lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x} = 1.$$

The graph of the function versus x confirms this result.

