

## Exercise 52

Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

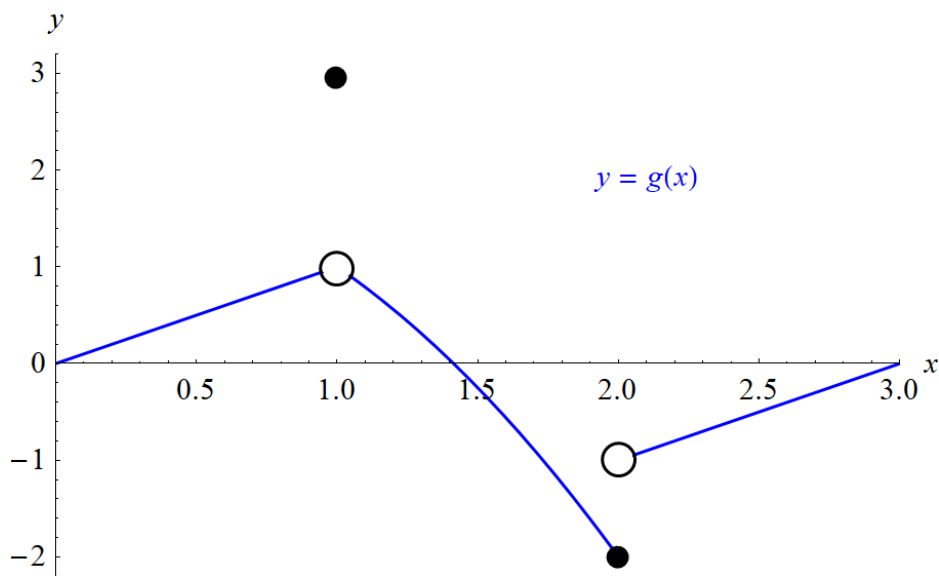
(a) Evaluate each of the following, if it exists.

$$\begin{array}{lll} \text{(i)} \quad \lim_{x \rightarrow 1^-} g(x) & \text{(ii)} \quad \lim_{x \rightarrow 1} g(x) & \text{(iii)} \quad g(1) \\ \text{(iv)} \quad \lim_{x \rightarrow 2^-} g(x) & \text{(v)} \quad \lim_{x \rightarrow 2^+} g(x) & \text{(vi)} \quad \lim_{x \rightarrow 2} g(x) \end{array}$$

(b) Sketch the graph of  $g$ .

### Solution

Below is a graph of  $g(x)$  versus  $x$ .



Use it to evaluate the limits.

$$\lim_{x \rightarrow 1^-} g(x) = 1$$

$$\lim_{x \rightarrow 1} g(x) = 1$$

$$g(1) = 3$$

$$\lim_{x \rightarrow 2^-} g(x) = -2$$

$$\lim_{x \rightarrow 2^+} g(x) = -1$$

$$\lim_{x \rightarrow 2} g(x) \text{ does not exist because } \lim_{x \rightarrow 2^-} g(x) \neq \lim_{x \rightarrow 2^+} g(x)$$