Exercise 7

For the limit

$$\lim_{x \to 2}(x^3 - 3x + 4) = 6$$

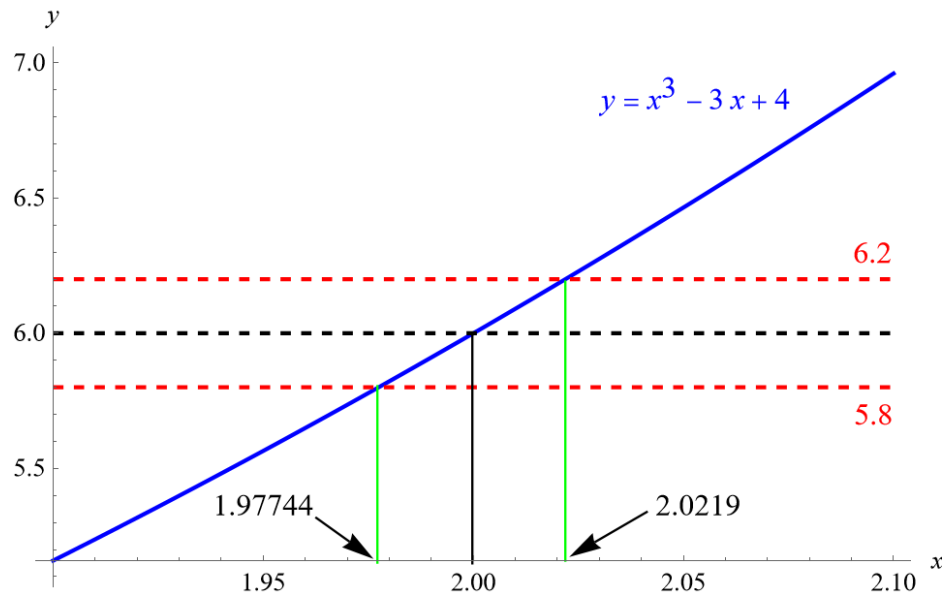
illustrate Definition 2 by finding values of δ that correspond to $\varepsilon = 0.2$ and $\varepsilon = 0.1$.

Solution

For $\varepsilon = 0.2$, Definition 2 says that this limit is equivalent to

$$\text{if} \qquad 0 < |x - 2| < \delta \qquad \text{then} \qquad |(x^3 - 3x + 4) - 6| < 0.2$$

for some positive δ.

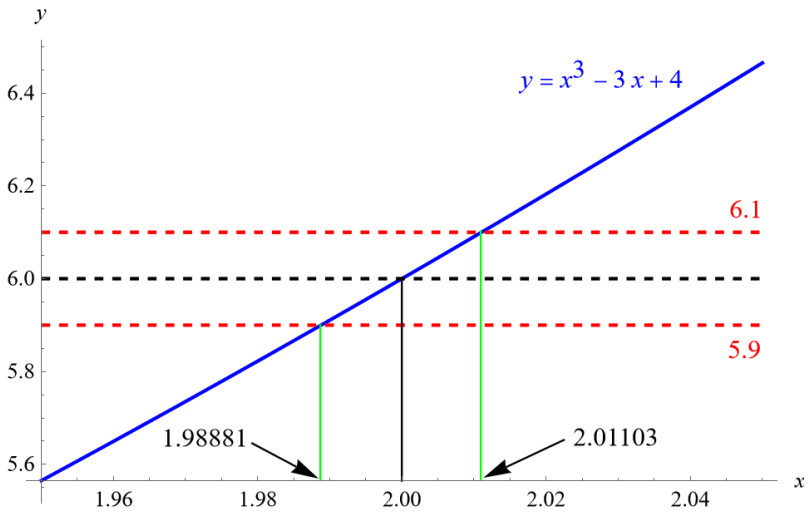


As long as δ is less than about $2.02190 - 2 \approx 0.02190$, the distance from 6 on the y-axis will be less than 0.2.

For $\varepsilon = 0.1$, Definition 2 says that this limit is equivalent to

$$\text{if} \qquad 0 < |x - 2| < \delta \qquad \text{then} \qquad |(x^3 - 3x + 4) - 6| < 0.1$$

for some positive δ.



As long as δ is less than about $2.01103 - 2 \approx 0.01103$, the distance from 6 on the y-axis will be less than 0.1.