Exercise 30

Explain, using Theorems 4, 5, 7, and 9, why the function is continuous at every number in its domain. State the domain.

$$B(x) = \frac{\tan x}{\sqrt{4 - x^2}}$$

Solution

In the numerator is a trigonometric function, and in the denominator is a root function. By Theorem 7 both of these are continuous at all numbers in their respective domains.

$$\tan x: \quad x \neq \frac{\pi}{2} + n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

$$\sqrt{4 - x^2}: \quad -2 \leq x \leq 2$$

And by Theorem 4 the ratio of these functions,

$$B(x) = \frac{\tan x}{\sqrt{4 - x^2}},$$

is continuous where the denominator is not zero.

$$\sqrt{4 - x^2} \neq 0$$

$$4 - x^2 \neq 0$$

$$(2 + x)(2 - x) \neq 0$$

$$x \neq -2 \quad \text{or} \quad x \neq 2$$

Therefore, combining these four conditions, the domain of $B(x)$ is

$$\left\{ x \,\middle|\, -2 < x < 2,\ x \neq -\frac{\pi}{2},\ x \neq \frac{\pi}{2} \right\}.$$