Exercise 16

Find the limit or show that it does not exist.

$$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1}$$

Solution

Multiply the numerator and denominator by the reciprocal of the highest power of x in the denominator.

$$\begin{aligned}
\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} &= \lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} \\
&= \lim_{x\to\infty} \frac{(1-x^2)\frac{1}{x^3}}{(x^3-x+1)\frac{1}{x^3}} \\
&= \lim_{x\to\infty} \frac{\frac{1}{x^3}-\frac{1}{x}}{1-\frac{1}{x^2}+\frac{1}{x^3}} \\
&= \frac{\lim\limits_{x\to\infty}\left(\frac{1}{x^3}-\frac{1}{x}\right)}{\lim\limits_{x\to\infty}\left(1-\frac{1}{x^2}+\frac{1}{x^3}\right)} \\
&= \frac{\lim\limits_{x\to\infty}\frac{1}{x^3}-\lim\limits_{x\to\infty}\frac{1}{x}}{\lim\limits_{x\to\infty}1-\lim\limits_{x\to\infty}\frac{1}{x^2}+\lim\limits_{x\to\infty}\frac{1}{x^3}} \\
&= \frac{0-0}{1-0+0} \\
&= 0
\end{aligned}$$