Exercise 28

Find the limit or show that it does not exist.

$$\lim_{x\to-\infty}\left(\sqrt{4x^2+3x}+2x\right)$$

Solution

Make the substitution, $u=-x$, so that as $x\to-\infty$, $u\to\infty$. Then multiply the numerator and denominator by the complex conjugate.

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{4x^2+3x}+2x\right) &= \lim_{u\to\infty}\left[\sqrt{4(-u)^2+3(-u)}+2(-u)\right] \\
&= \lim_{u\to\infty}\left(\sqrt{4u^2-3u}-2u\right) \\
&= \lim_{u\to\infty}\left(\sqrt{4u^2-3u}-2u\right)\cdot\frac{\sqrt{4u^2-3u}+2u}{\sqrt{4u^2-3u}+2u} \\
&= \lim_{u\to\infty}\frac{\left(\sqrt{4u^2-3u}-2u\right)\left(\sqrt{4u^2-3u}+2u\right)}{\sqrt{4u^2-3u}+2u} \\
&= \lim_{u\to\infty}\frac{(4u^2-3u)-(4u^2)}{\sqrt{4u^2-3u}+2u} \\
&= \lim_{u\to\infty}\frac{-3u}{\sqrt{4u^2-3u}+2u} \\
&= \lim_{u\to\infty}\frac{-3u}{\sqrt{u^2\left(4-\frac{3}{u}\right)}+2u} \\
&= \lim_{u\to\infty}\frac{-3u}{u\sqrt{4-\frac{3}{u}}+2u} \\
&= \lim_{u\to\infty}\frac{-3u}{u\left(\sqrt{4-\frac{3}{u}}+2\right)} \\
&= \lim_{u\to\infty}\frac{-3}{\sqrt{4-\frac{3}{u}}+2} \\
&= \frac{-3}{\sqrt{4-0}+2} \\
&= -\frac{3}{4}
\end{aligned}$$