Exercise 9

(a) Find the slope of the tangent to the curve $y = 3 + 4x^2 - 2x^3$ at the point where $x = a$.

(b) Find equations of the tangent lines at the points $(1, 5)$ and $(2, 3)$.

(c) Graph the curve and both tangents on a common screen.

Solution

Part (a)

Start by finding the slope of the tangent line to the curve at $x = a$.

$$\begin{aligned} m = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} &= \lim_{x\to a} \frac{(3 + 4x^2 - 2x^3) - (3 + 4a^2 - 2a^3)}{x - a} \\ &= \lim_{x\to a} \frac{4x^2 - 2x^3 - 4a^2 + 2a^3}{x - a} \\ &= \lim_{x\to a} \frac{4(x^2 - a^2) - 2(x^3 - a^3)}{x - a} \\ &= \lim_{x\to a} \frac{4(x + a)(x - a) - 2(x - a)(x^2 + ax + a^2))}{x - a} \\ &= \lim_{x\to a} [4(x + a) - 2(x^2 + ax + a^2)] \\ &= 4(a + a) - 2(a^2 + a \cdot a + a^2) \\ &= 4(2a) - 2(3a^2) \\ &= 8a - 6a^2 \end{aligned}$$

Part (b)

For the point $(1, 5)$, the slope is

$$m = 8(1) - 6(1)^2 = 2.$$

The equation of the line is then

$$y - 5 = 2(x - 1)$$

$$y - 5 = 2x - 2$$

$$y = 2x + 3.$$

For the point $(2, 3)$, the slope is

$$m = 8(2) - 6(2)^2 = -8.$$

The equation of the line is then

$$y - 3 = -8(x - 2)$$

$$y - 3 = -8x + 16$$

$$y = -8x + 19.$$

Part (c)

Below is a graph of $y = 3 + 4x^2 - 2x^3$ versus x along with the tangent lines at $x = 1$ and $x = 2$.