

Exercise 23

If $2x - 1 \leq f(x) \leq x^2$ for $0 < x < 3$, find $\lim_{x \rightarrow 1} f(x)$.

Solution

Assume the given inequality is true.

$$2x - 1 \leq f(x) \leq x^2$$

Take the limit of all sides of the inequality as $x \rightarrow 1$.

$$\lim_{x \rightarrow 1} (2x - 1) \leq \lim_{x \rightarrow 1} f(x) \leq \lim_{x \rightarrow 1} x^2$$

Evaluate the limits.

$$2(1) - 1 \leq \lim_{x \rightarrow 1} f(x) \leq (1)^2$$

Simplify the left and right sides.

$$1 \leq \lim_{x \rightarrow 1} f(x) \leq 1$$

Therefore,

$$\lim_{x \rightarrow 1} f(x) = 1.$$