Exercise 70

Find a parabola with equation $y = ax^2 + bx + c$ that has slope 4 at $x = 1$, slope -8 at $x = -1$, and passes through the point $(2, 15)$.

Solution

Take the derivative of the given function.

$$\begin{aligned} y' &= \frac{d}{dx}(ax^2 + bx + c) \\ &= \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c) \\ &= a\frac{d}{dx}(x^2) + b\frac{d}{dx}(x) + \frac{d}{dx}(c) \\ &= a(2x) + b(1) + (0) \\ &= 2ax + b \end{aligned}$$

The graph of y has slope 4 at $x = 1$ and slope -8 at $x = -1$.

$$2a(1) + b = 4$$

$$2a(-1) + b = -8$$

Solve this system of equations for a and b.

$$a = 3 \qquad b = -2$$

Consequently,

$$y = 3x^2 - 2x + c.$$

Use the fact that $y = 15$ when $x = 2$ to determine c.

$$y(2) = 3(2)^2 - 2(2) + c = 15$$

Solve for c.

$$c = 7$$

Therefore, the parabola that has slope 4 at $x = 1$, slope -8 at $x = -1$, and passes through the point $(2, 15)$ is

$$y = 3x^2 - 2x + 7.$$