Exercise 43

Find the derivative. Simplify where possible.

$$y = x\sinh^{-1}(x/3) - \sqrt{9+x^2}$$

Solution

Take the derivative using the chain and product rules.

$$\begin{aligned}
y' &= \frac{d}{dx}\left[x\sinh^{-1}\left(\frac{x}{3}\right) - \sqrt{9+x^2}\right] \\
&= \frac{d}{dx}\left[x\sinh^{-1}\left(\frac{x}{3}\right)\right] - \frac{d}{dx}\left(\sqrt{9+x^2}\right) \\
&= \left[\frac{d}{dx}(x)\right]\sinh^{-1}\left(\frac{x}{3}\right) + x\left[\frac{d}{dx}\sinh^{-1}\left(\frac{x}{3}\right)\right] - \frac{1}{2}(9+x^2)^{-1/2}\cdot\frac{d}{dx}(9+x^2) \\
&= (1)\sinh^{-1}\left(\frac{x}{3}\right) + x\left[\frac{1}{\sqrt{1+\left(\frac{x}{3}\right)^2}}\cdot\frac{d}{dx}\left(\frac{x}{3}\right)\right] - \frac{1}{2}(9+x^2)^{-1/2}\cdot(2x) \\
&= \sinh^{-1}\left(\frac{x}{3}\right) + x\left[\frac{1}{\sqrt{1+\frac{x^2}{9}}}\cdot\left(\frac{1}{3}\right)\right] - \frac{x}{\sqrt{9+x^2}} \\
&= \sinh^{-1}\left(\frac{x}{3}\right) + \frac{x}{\sqrt{9+x^2}} - \frac{x}{\sqrt{9+x^2}} \\
&= \sinh^{-1}\left(\frac{x}{3}\right)
\end{aligned}$$