## Exercise 50

Let $P(x)=F(x) G(x)$ and $Q(x)=F(x) / G(x)$, where $F$ and $G$ are the functions whose graphs are shown.
(a) Find $P^{\prime}(2)$.
(b) Find $Q^{\prime}(7)$.


## Solution

Evaluate the derivative of $P(x)$ using the product rule.

$$
P^{\prime}(x)=F^{\prime}(x) G(x)+F(x) G^{\prime}(x)
$$

Evaluate the derivative of $Q(x)$ using the quotient rule.

$$
Q^{\prime}(x)=\frac{F^{\prime}(x) G(x)-G^{\prime}(x) F(x)}{[G(x)]^{2}}
$$

At $x=2$, the slope of $F$ is 0 and the slope of $G$ is $1 / 2: F^{\prime}(2)=0$ and $G^{\prime}(2)=1 / 2$. Use this information to evaluate $P^{\prime}(2)$.

$$
P^{\prime}(2)=F^{\prime}(2) G(2)+F(2) G^{\prime}(2)=(0)(2)+(3)\left(\frac{1}{2}\right)=\frac{3}{2}
$$

At $x=7$, the slope of $F$ is $1 / 4$ and the slope of $G$ is $-2 / 3: F^{\prime}(7)=1 / 4$ and $G^{\prime}(7)=-2 / 3$. Use this information to evaluate $Q^{\prime}(7)$.

$$
Q^{\prime}(7)=\frac{F^{\prime}(7) G(7)-G^{\prime}(7) F(7)}{[G(7)]^{2}}=\frac{\left(\frac{1}{4}\right)(1)-\left(-\frac{2}{3}\right)(5)}{1^{2}}=\frac{43}{12}
$$

