Exercise 64

(a) If g is differentiable, the **Reciprocal Rule** says that

$$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{[g(x)]^2}$$

Use the Quotient Rule to prove the Reciprocal Rule.

(b) Use the Reciprocal Rule to differentiate the function in Exercise 16.

(c) Use the Reciprocal Rule to verify that the Power Rule is valid for negative integers, that is,

$$\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$$

for all positive integers n.

Solution

Part (a)

Use the quotient rule to prove the reciprocal rule.

$$\begin{aligned}\frac{d}{dx}\left[\frac{1}{g(x)}\right] &= \frac{\left[\frac{d}{dx}(1)\right]g(x) - g'(x)(1)}{[g(x)]^2} \\ &= \frac{(0)g(x) - g'(x)(1)}{[g(x)]^2} \\ &= -\frac{g'(x)}{[g(x)]^2}\end{aligned}$$

Part (b)

The function to differentiate in Exercise 16 is

$$y = \frac{1}{t^3 + 2t^2 - 1}.$$

Here $g(t) = t^3 + 2t^2 - 1$, so $g'(t) = 3t^2 + 4t$. Therefore, by the reciprocal rule,

$$y' = -\frac{3t^2 + 4t}{(t^3 + 2t^2 - 1)^2}.$$

This result is in agreement with the one in Exercise 16.

Part (c)

Suppose that n is a positive integer.

$$\frac{d}{dx}(x^{-n}) = \frac{d}{dx}\left(\frac{1}{x^n}\right)$$

Here $g(x) = x^n$, so $g'(x) = nx^{n-1}$. Therefore, by the reciprocal rule,

$$y' = -\frac{nx^{n-1}}{(x^n)^2} = -nx^{n-1-2n} = -nx^{-n-1}$$

for all positive integers n.