

Exercise 17

Find the derivative of the function.

$$f(x) = (2x - 3)^4(x^2 + x + 1)^5$$

Solution

Take the derivative using the product rule and the chain rule.

$$\begin{aligned} f'(x) &= \frac{df}{dx} = \frac{d}{dx}[(2x - 3)^4(x^2 + x + 1)^5] \\ &= \left[\frac{d}{dx}(2x - 3)^4 \right] (x^2 + x + 1)^5 + (2x - 3)^4 \left[\frac{d}{dx}(x^2 + x + 1)^5 \right] \\ &= \left[4(2x - 3)^3 \cdot \frac{d}{dx}(2x - 3) \right] (x^2 + x + 1)^5 + (2x - 3)^4 \left[5(x^2 + x + 1)^4 \cdot \frac{d}{dx}(x^2 + x + 1) \right] \\ &= [4(2x - 3)^3 \cdot (2)] (x^2 + x + 1)^5 + (2x - 3)^4 [5(x^2 + x + 1)^4 \cdot (2x + 1)] \\ &= 8(2x - 3)^3(x^2 + x + 1)^5 + 5(2x + 1)(2x - 3)^4(x^2 + x + 1)^4 \\ &= (2x - 3)^3(x^2 + x + 1)^4[8(x^2 + x + 1) + 5(2x + 1)(2x - 3)] \\ &= (2x - 3)^3(x^2 + x + 1)^4(28x^2 - 12x - 7) \end{aligned}$$