

Exercise 39

Find the derivative of the function.

$$f(t) = \tan(\sec(\cos t))$$

Solution

Take the derivative using the chain rule twice.

$$\begin{aligned} f'(t) &= \frac{df}{dt} = \frac{d}{dt} \{\tan[\sec(\cos t)]\} \\ &= \sec^2[\sec(\cos t)] \cdot \frac{d}{dt}[\sec(\cos t)] \\ &= \sec^2[\sec(\cos t)] \cdot [\sec(\cos t) \tan(\cos t)] \cdot \frac{d}{dt}(\cos t) \\ &= \sec^2[\sec(\cos t)] \cdot [\sec(\cos t) \tan(\cos t)] \cdot (-\sin t) \\ &= -\sec^2[\sec(\cos t)] \sec(\cos t) \tan(\cos t) \sin t \end{aligned}$$