

Exercise 50

Find y' and y'' .

$$y = e^{e^x}$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned}y' &= \frac{dy}{dx} = \frac{d}{dx}(e^{e^x}) \\&= e^{e^x} \cdot \frac{d}{dx}(e^x) \\&= e^{e^x} \cdot (e^x) \\&= e^{e^x+x}\end{aligned}$$

Take another derivative.

$$\begin{aligned}y'' &= \frac{d}{dx}(y') = \frac{d}{dx}(e^{e^x+x}) \\&= e^{e^x+x} \cdot \frac{d}{dx}(e^x + x) \\&= e^{e^x+x} \cdot (e^x + 1) \\&= (e^x + 1)e^{e^x+x}\end{aligned}$$