Exercise 27

Differentiate f and find the domain of f.

$$f(x) = \frac{x}{1 - \ln(x-1)}$$

Solution

Recognize that the denominator of a rational function cannot be zero, and only the logarithm of a positive number can be taken.

$$1 - \ln(x-1) \neq 0 \quad \text{and} \quad x - 1 > 0$$

$$\ln(x-1) \neq 1 \quad \text{and} \quad x > 1$$

$$x - 1 \neq e^1 \quad \text{and} \quad x > 1$$

$$x \neq 1 + e \quad \text{and} \quad x > 1$$

Therefore, the domain of the function is

$$(1, 1+e) \cup (1+e, \infty).$$

Take the derivative of the function with respect to x by using the chain and quotient rules.

$$\begin{aligned}
f'(x) &= \frac{d}{dx}\left[\frac{x}{1-\ln(x-1)}\right] \\
&= \frac{\left[\frac{d}{dx}(x)\right][1-\ln(x-1)] - \left\{\frac{d}{dx}[1-\ln(x-1)]\right\}x}{[1-\ln(x-1)]^2} \\
&= \frac{(1)[1-\ln(x-1)] - \left[-\frac{1}{x-1}\cdot\frac{d}{dx}(x-1)\right]x}{[1-\ln(x-1)]^2} \\
&= \frac{[1-\ln(x-1)] - \left[-\frac{1}{x-1}\cdot(1)\right]x}{[1-\ln(x-1)]^2} \\
&= \frac{1-\ln(x-1)+\frac{x}{x-1}}{[1-\ln(x-1)]^2} \times \frac{x-1}{x-1} \\
&= \frac{(x-1)-(x-1)\ln(x-1)+x}{[1-\ln(x-1)]^2(x-1)} \\
&= \frac{2x-1-(x-1)\ln(x-1)}{(x-1)[1-\ln(x-1)]^2}
\end{aligned}$$