

Exercise 39

Use logarithmic differentiation to find the derivative of the function.

$$y = (x^2 + 2)^2(x^4 + 4)^4$$

Solution

Take the natural logarithm of both sides and use the properties of logarithms to simplify the right side.

$$\begin{aligned}\ln y &= \ln(x^2 + 2)^2(x^4 + 4)^4 \\ &= \ln(x^2 + 2)^2 + \ln(x^4 + 4)^4 \\ &= 2\ln(x^2 + 2) + 4\ln(x^4 + 4)\end{aligned}$$

Differentiate both sides with respect to x .

$$\begin{aligned}\frac{d}{dx}(\ln y) &= \frac{d}{dx}[2\ln(x^2 + 2) + 4\ln(x^4 + 4)] \\ \frac{1}{y} \cdot \frac{d}{dx}(y) &= \frac{2}{x^2 + 2} \cdot \frac{d}{dx}(x^2 + 2) + \frac{4}{x^4 + 4} \cdot \frac{d}{dx}(x^4 + 4) \\ \frac{1}{y} \cdot \frac{dy}{dx} &= \frac{2}{x^2 + 2} \cdot (2x) + \frac{4}{x^4 + 4} \cdot (4x^3) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{4x}{x^2 + 2} + \frac{16x^3}{x^4 + 4} \\ \frac{dy}{dx} &= y \left[\frac{4x}{x^2 + 2} + \frac{16x^3}{x^4 + 4} \right] \\ &= (x^2 + 2)^2(x^4 + 4)^4 \left[\frac{4x}{x^2 + 2} + \frac{16x^3}{x^4 + 4} \right] \\ &= 4x(x^2 + 2)(x^4 + 4)^4 + 16x^3(x^2 + 2)^2(x^4 + 4)^3 \\ &= 4x(x^2 + 2)(x^4 + 4)^3[(x^4 + 4) + 4x^2(x^2 + 2)] \\ &= 4x(x^2 + 2)(x^4 + 4)^3(5x^4 + 8x^2 + 4)\end{aligned}$$