Exercise 1

A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.

(b) What is the velocity after 1 second?

(c) When is the particle at rest?

(d) When is the particle moving in the positive direction?

(e) Find the total distance traveled during the first 6 seconds.

(f) Draw a diagram like Figure 2 to illustrate the motion of the particle.

(g) Find the acceleration at time t and after 1 second.

(h) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 6$.

(i) When is the particle speeding up? When is it slowing down?

$$f(t) = t^3 - 8t^2 + 24t$$

Solution

Part (a)

To find the velocity, take the derivative of the position function.

$$\begin{aligned} v(t) &= \frac{ds}{dt} \\ &= \frac{d}{dt}(t^3 - 8t^2 + 24t) \\ &= 3t^2 - 16t + 24 \end{aligned}$$

Part (b)

The velocity after 1 second has elapsed is

$$v(1) = 3(1)^2 - 16(1) + 24 = 11 \ \frac{\text{feet}}{\text{second}}.$$

Part (c)

To find when the particle is at rest, set the velocity function equal to zero and solve the equation for t.

$$v(t) = 0$$

$$3t^2 - 16t + 24 = 0$$

$$t = \frac{16 \pm \sqrt{16^2 - 4(3)(24)}}{2(3)}$$

$$t = \frac{16 \pm \sqrt{-32}}{6}$$

$$t = \frac{16 \pm 4i\sqrt{2}}{6}$$

Since no real values of t satisfy the equation, the particle is never at rest.

Part (d)

Since $v(t) = 3t^2 - 16t + 24$ is a continuous function, $v(0) = 24$ is positive, and $v(t)$ never crosses the t-axis, $v(t)$ is always positive. This means the particle is always moving in the positive direction.

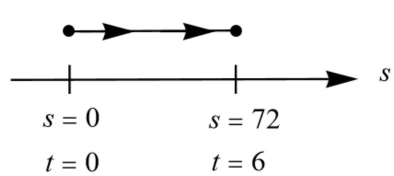
Part (e)

The total distance traveled is

$$\begin{aligned} s(6) - s(0) &= [(6)^3 - 8(6)^2 + 24(6)] - [(0)^3 - 8(0)^2 + 24(0)] \\ &= 72 - 0 \\ &= 72 \text{ feet.} \end{aligned}$$

Part (f)

Below is an illustration of the particle's motion from $t = 0$ to $t = 6$.



Part (g)

Calculate the derivative of the velocity to get the acceleration.

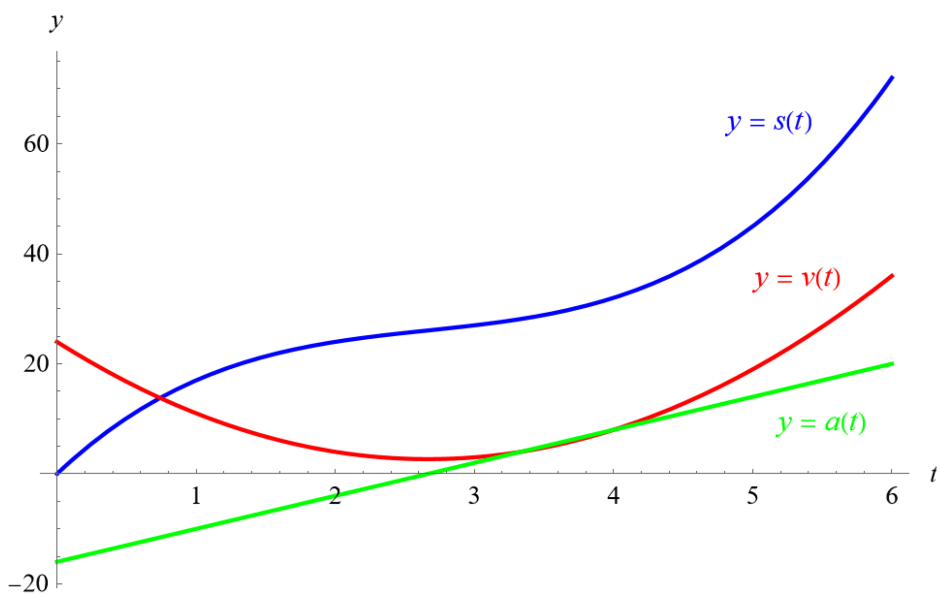
$$\begin{aligned} a(t) &= \frac{dv}{dt} \\ &= \frac{d}{dt}(3t^2 - 16t + 24) \\ &= 6t - 16 \end{aligned}$$

The acceleration after 1 second is

$$a(1) = 6(1) - 16 = -10 \ \frac{\text{feet}}{\text{second}^2}.$$

Part (h)

Below is a plot of the position, velocity, and acceleration versus time for $0 \le t \le 6$.



Part (i)

The particle is speeding up when

$$a(t) > 0$$

$$6t - 16 > 0$$

$$6t > 16$$

$$t > \frac{8}{3} \text{ seconds},$$

and the particle is slowing down when

$$a(t) < 0$$

$$6t - 16 < 0$$

$$6t < 16$$

$$t < \frac{8}{3} \text{ seconds.}$$