Exercise 13

Dinosaur fossils are often dated by using an element other than carbon, such as potassium-40, that has a longer half-life (in this case, approximately 1.25 billion years). Suppose the minimum detectable amount is 0.1% and a dinosaur is dated with ^{40}K to be 68 million years old. Is such a dating possible? In other words, what is the maximum age of a fossil that we could date using ^{40}K?

Solution

Assume that the rate of mass decay is proportional to the amount of mass remaining at any given time.

$$\frac{dm}{dt} \propto -m$$

There's a minus sign here because mass is being lost as time increases. Change the proportionality to an equation by introducing a (positive) constant k.

$$\frac{dm}{dt} = -km$$

Divide both sides by m.

$$\frac{1}{m}\frac{dm}{dt} = -k$$

Rewrite the left side by using the chain rule.

$$\frac{d}{dt}\ln m = -k$$

The function you have to differentiate to get $-k$ is $-kt + C$, where C is any constant.

$$\ln m = -kt + C$$

Exponentiate both sides.

$$e^{\ln m} = e^{-kt+C}$$

$$m(t) = e^C e^{-kt}$$

Use a new constant m_0 for e^C.

$$m(t) = m_0 e^{-kt} \tag{1}$$

The half-life is defined as the amount of time it takes for a sample to decay to half its mass, so set $m(1.25 \times 10^9) = m_0/2$ and solve the equation for k.

$$m(1.25 \times 10^9) = \frac{m_0}{2}$$

$$m_0 e^{-k(1.25\times 10^9)} = \frac{m_0}{2}$$

$$e^{-1.25\times 10^9 k} = \frac{1}{2}$$

$$\ln e^{-1.25\times 10^9 k} = \ln \frac{1}{2}$$

$$(-1.25 \times 10^9 k)\ln e = -\ln 2$$

$$k = \frac{\ln 2}{1.25 \times 10^9} \approx 5.54518 \times 10^{-10} \text{ year}^{-1}$$

As a result, equation (1) becomes

$$\begin{aligned} m(t) &= m_0 e^{-\left(\frac{\ln 2}{1.25\times 10^9}\right)t} \\ &= m_0 e^{\ln 2^{-t/(1.25\times 10^9)}} \\ &= m_0 (2)^{-t/(1.25\times 10^9)}. \end{aligned}$$

To find how long it takes for the ^{40}K to reduce to 0.1% of its original amount, set $m(t) = 0.001 m_0$ and solve the equation for t.

$$m(t) = 0.001 m_0$$

$$m_0(2)^{-t/(1.25\times 10^9)} = 0.001 m_0$$

$$2^{-t/(1.25\times 10^9)} = 0.001$$

$$\ln 2^{-t/(1.25\times 10^9)} = \ln 0.001$$

$$\left(-\frac{t}{1.25 \times 10^9}\right)\ln 2 = \ln 0.001$$

$$t = -\frac{1.25 \times 10^9 \ln 0.001}{\ln 2} \approx 1.24572 \times 10^{10} \text{ years}$$

This is the maximum age of a fossil that can be dated with ^{40}K if the minimum detectable amount is 0.1%. A dinosaur fossil that's 6.8×10^7 years old can be dated with this isotope.