Exercise 14

(a) What quantities are given in the problem?

(b) What is the unknown?

(c) Draw a picture of the situation for any time t.

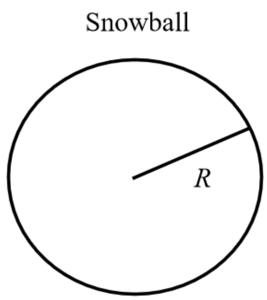
(d) Write an equation that relates the quantities.

(e) Finish solving the problem.

If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

Solution

The rate that surface area is increasing ($dS/dt = -1$ cm^2/min) is given. The rate that D, the snowball's diameter, is increasing is unknown.



The relationship between S and radius R is given by

$$S = 4\pi R^2.$$

Write it in terms of diameter.

$$S = 4\pi \left(\frac{D}{2}\right)^2$$

$$S = \pi D^2$$

Differentiate both sides with respect to time.

$$\frac{dS}{dt} = \frac{d}{dt}(\pi D^2) = \left(2\pi D \cdot \frac{dD}{dt}\right)$$

Solve for dD/dt, the rate that diameter increases with respect to time.

$$\frac{dD}{dt} = \frac{1}{2\pi D}\frac{dS}{dt}$$

Therefore, the rate that D is increasing when $D = 10$ is

$$\left.\frac{dD}{dt}\right|_{D=10} = \frac{1}{2\pi(10 \text{ cm})}\left(-1\,\frac{\text{cm}^2}{\text{min}}\right) = -\frac{1}{20\pi}\,\frac{\text{cm}}{\text{min}}.$$