Exercise 15

(a) What quantities are given in the problem?

(b) What is the unknown?

(c) Draw a picture of the situation for any time t.

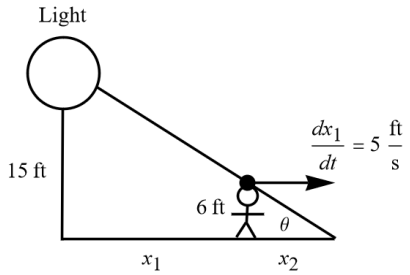
(d) Write an equation that relates the quantities.

(e) Finish solving the problem.

A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

Solution

The man's walking speed ($dx_1/dt = 5$ ft/s) is given. dx_1/dt represents the man's speed with respect to the pole, and dx_2/dt represents the shadow's tip speed with respect to the man. To get the shadow's tip speed with respect to the pole (the unknown), add dx_1/dt and dx_2/dt together.



Use trigonometry to relate the sides of the triangles.

$$\tan\theta = \frac{6}{x_2} = \frac{15}{x_1 + x_2}$$

$$6(x_1 + x_2) = 15x_2$$

$$6x_1 + 6x_2 = 15x_2$$

$$6x_1 = 9x_2$$

$$x_2 = \frac{2}{3}x_1$$

Differentiate both sides with respect to t.

$$\frac{dx_2}{dt} = \frac{2}{3}\frac{dx_1}{dt}$$

Therefore, the shadow's tip speed with respect to the pole is

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = \frac{dx_1}{dt} + \frac{2}{3}\frac{dx_1}{dt} = \frac{5}{3}\frac{dx_1}{dt} = \frac{5}{3}\left(5\ \frac{\text{ft}}{\text{s}}\right) = \frac{25}{3}\ \frac{\text{ft}}{\text{s}}.$$