# Exercise 36

A faucet is filling a hemispherical basin of diameter 60 cm with water at a rate of 2 L/min. Find the rate at which the water is rising in the basin when it is half full. [Use the following facts: 1 L is 1000 $\text{cm}^3$. The volume of the portion of a sphere with radius $r$ from the bottom to a height $h$ is $V = \pi(rh^2 - \frac{1}{3}h^3)$, as we will show in Chapter 6.]

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### Solution

Start with the formula for the volume of water up to a height $h$ in a hemispherical basin with radius 30 cm.

$$V = \pi\left(30h^2 - \frac{1}{3}h^3\right)$$

$$= 30\pi h^2 - \frac{\pi}{3}h^3$$

Take the derivative of both sides with respect to $t$ by using the chain rule.

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(30\pi h^2 - \frac{\pi}{3}h^3\right)$$

$$\frac{dV}{dt} = 30\pi(2h)\cdot\frac{dh}{dt} - \frac{\pi}{3}(3h^2)\cdot\frac{dh}{dt}$$

$$2000\,\frac{\text{cm}^3}{\text{min}} = (60\pi h - \pi h^2)\frac{dh}{dt}$$

Solve for $dh/dt$.

$$\frac{dh}{dt} = \frac{2000}{60\pi h - \pi h^2}$$

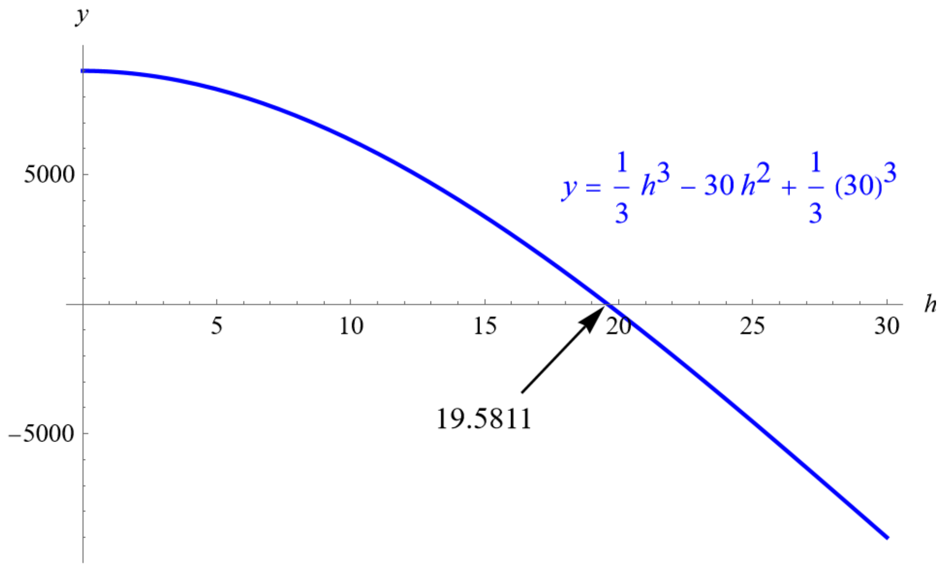
The aim now is to find the value of $h$ for which the basin is half full. Note that the volume of a hemisphere with radius $r$ is $(2/3)\pi r^3$.

$$V = \pi(rh^2 - \frac{1}{3}h^3) \quad \Rightarrow \quad \frac{1}{2}\left[\frac{2}{3}\pi(30)^3\right] = \pi\left(30h^2 - \frac{1}{3}h^3\right)$$

Divide both sides by $\pi$ and bring all terms to the left side.

$$\frac{1}{3}h^3 - 30h^2 + \frac{1}{3}(30)^3 = 0$$

Obtain the value of $h$ that satisfies this equation by plotting the function on the left side and seeing where it crosses the horizontal axis.




Therefore, the rate at which the water is rising when the basin is half full is

$$\left.\frac{dh}{dt}\right|_{h\approx 19.5811} \approx \frac{2000}{60\pi(19.5811) - \pi(19.5811)^2} \approx 0.804375\ \frac{\text{cm}}{\text{min}}.$$