Exercise 37

Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant the volume is 600 cm^3, the pressure is 150 kPa, and the pressure is increasing at a rate of 20 kPa/min. At what rate is the volume decreasing at this instant?

Solution

Solve the given formula for the volume.

$$V = \frac{C}{P}$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\begin{aligned}
\frac{d}{dt}(V) &= \frac{d}{dt}\left(\frac{C}{P}\right) \\
\frac{dV}{dt} &= -\frac{C}{P^2} \cdot \frac{dP}{dt} \\
&= -\left(\frac{C}{P}\right)\frac{1}{P}\frac{dP}{dt} \\
&= -(V)\frac{1}{P}\frac{dP}{dt} \\
&= -\frac{V}{P}\frac{dP}{dt}.
\end{aligned}$$

Therefore, at the instant that the volume is 600 cm^3, the pressure is 150 kPa, and the pressure is increasing at a rate of 20 kPa/min, the rate of change of the volume is

$$\left.\frac{dV}{dt}\right|_{\substack{V=600\\P=150}} = -\frac{600}{150}(20) = -80\ \frac{\text{cm}^3}{\text{min}}.$$

The minus sign indicates that the volume decreases as time goes on.