Exercise 38

When air expands adiabatically (without gaining or losing heat), its pressure P and volume V are related by the equation $PV^{1.4} = C$, where C is a constant. Suppose that at a certain instant the volume is 400 cm^3 and the pressure is 80 kPa and is decreasing at a rate of 10 kPa/min. At what rate is the volume increasing at this instant?

Solution

Solve the given formula for the volume.

$$V^{1.4} = \frac{C}{P}$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\frac{d}{dt}(V^{1.4}) = \frac{d}{dt}\left(\frac{C}{P}\right)$$

$$\begin{aligned} 1.4V^{0.4} \cdot \frac{dV}{dt} &= -\frac{C}{P^2} \cdot \frac{dP}{dt} \\ &= -\left(\frac{C}{P}\right)\frac{1}{P}\frac{dP}{dt} \\ &= -(V^{1.4})\frac{1}{P}\frac{dP}{dt} \end{aligned}$$

Solve for dV/dt.

$$\frac{dV}{dt} = -\frac{V}{1.4P}\frac{dP}{dt}.$$

Therefore, at the instant that the volume is 400 cm^3, the pressure is 80 kPa, and the pressure is decreasing at a rate of 10 kPa/min, the rate of change of the volume is

$$\left.\frac{dV}{dt}\right|_{\substack{V=400\\P=80}} = -\frac{400}{1.4(80)}(-10) = \frac{250}{7}\ \frac{\text{cm}^3}{\text{min}} \approx 35.7143\ \frac{\text{cm}^3}{\text{min}}.$$