

**Exercise 19**

Calculate  $y'$ .

$$y = \tan\left(\frac{t}{1+t^2}\right)$$

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**Solution**

Calculate  $y'$  by using the chain and quotient rules.

$$\begin{aligned}y' &= \frac{d}{dt} \tan\left(\frac{t}{1+t^2}\right) \\&= \sec^2\left(\frac{t}{1+t^2}\right) \cdot \frac{d}{dt}\left(\frac{t}{1+t^2}\right) \\&= \sec^2\left(\frac{t}{1+t^2}\right) \cdot \frac{\left[\frac{d}{dt}(t)\right](1+t^2) - \left[\frac{d}{dt}(1+t^2)\right](t)}{(1+t^2)^2} \\&= \sec^2\left(\frac{t}{1+t^2}\right) \cdot \frac{(1)(1+t^2) - (2t)(t)}{(1+t^2)^2} \\&= \sec^2\left(\frac{t}{1+t^2}\right) \cdot \frac{1-t^2}{(1+t^2)^2} \\&= \frac{1-t^2}{(1+t^2)^2} \sec^2\left(\frac{t}{1+t^2}\right)\end{aligned}$$