

Exercise 51

If $f(t) = \sqrt{4t + 1}$, find $f''(2)$.

Solution

Calculate the first derivative of $f(t)$.

$$\begin{aligned} f'(t) &= \frac{d}{dt} \sqrt{4t + 1} \\ &= \frac{1}{2} (4t + 1)^{-1/2} \cdot \frac{d}{dt} (4t + 1) \\ &= \frac{1}{2} (4t + 1)^{-1/2} \cdot (4) \\ &= 2(4t + 1)^{-1/2} \end{aligned}$$

Calculate the second derivative of $f(t)$.

$$\begin{aligned} f''(t) &= \frac{d}{dt} [f'(t)] \\ &= \frac{d}{dt} [2(4t + 1)^{-1/2}] \\ &= 2 \frac{d}{dt} (4t + 1)^{-1/2} \\ &= 2 \left(-\frac{1}{2} \right) (4t + 1)^{-3/2} \cdot \frac{d}{dt} (4t + 1) \\ &= 2 \left(-\frac{1}{2} \right) (4t + 1)^{-3/2} \cdot (4) \\ &= -4(4t + 1)^{-3/2} \\ &= -\frac{4}{(\sqrt{4t + 1})^3} \end{aligned}$$

Plug in $t = 2$ to find $f''(2)$.

$$f''(2) = -\frac{4}{[\sqrt{4(2) + 1}]^3} = -\frac{4}{(3)^3} = -\frac{4}{27}$$