Exercise 12

Verify by direct substitution that

$$u_n(x, y) = \sin nx \sinh ny$$

is a solution of $u_{xx} + u_{yy} = 0$ for every $n > 0$.

Solution

Suppose n is some number greater than zero. Then

$$\begin{aligned} u_x &= n \cos nx \sinh ny \\ u_{xx} &= -n^2 \sin nx \sinh ny \\ u_y &= n \sin nx \cosh ny \\ u_{yy} &= n^2 \sin nx \sinh ny \end{aligned}$$

Substituting the expressions,

$$\begin{aligned} u_{xx} + u_{yy} &= -n^2 \sin nx \sinh ny + n^2 \sin nx \sinh ny \\ &= (-n^2 + n^2) \sin nx \sinh ny \\ &= (0) \sin nx \sinh ny \\ &= 0 \end{aligned}$$

Therefore, $u = u_n(x, y) = \sin nx \sinh ny$ is a solution of the PDE for every $n > 0$.