Exercise 4

Check that (7) indeed solves (4).

Solution

The problem is asking us to check that the solution in (7) satisfies the PDE in (4).

$$u_x + yu_y = 0 \tag{4}$$

$$u(x, y) = f(e^{-x}y) \tag{7}$$

Evaluating the first derivatives in x and y gives

$$\begin{aligned} u_x &= (ye^{-x})_x f' = -ye^{-x}f' \\ u_y &= (ye^{-x})_y f' = e^{-x}f'. \end{aligned}$$

Multiplying u_y by y gives $ye^{-x}f'$. Therefore,

$$u_x + yu_y = -ye^{-x}f' + ye^{-x}f' = 0,$$

and (7) indeed solves (4).