Exercise 5

In Exercise 1.3.4, find the boundary condition if the particles lie above an impermeable horizontal plane $z = a$.

Solution

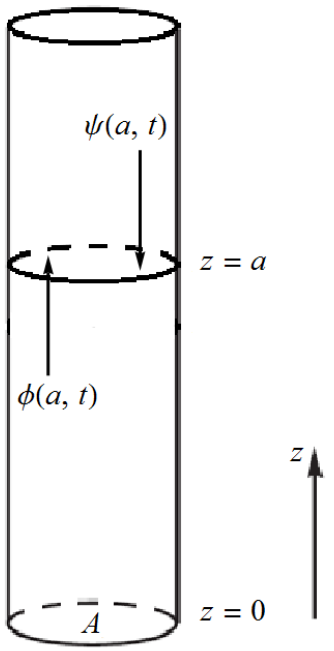

Figure 1: Schematic of the medium with equal and opposite fluxes, ϕ and ψ, due to diffusion and gravity, respectively.

The fact that the plane $z = a$ is impermeable means that no mass can cross it, i.e. the mass flux at this point must be 0. In this exercise there are two forces at work: diffusion (pushing the particles up) and gravity (pulling the particles down). The mass flux due to diffusion is given by Fick's first law:

$$\phi = -k\frac{\partial u}{\partial z}.$$

The mass flux due to gravity is given by the product of the speed the particles fall under its influence and the concentration.

$$\psi = Vu.$$

In order for no mass to flow, these opposing fluxes must be equal at this point. Therefore, the boundary condition is

$$-k\frac{\partial u}{\partial z}(a,t) = Vu(a,t).$$