

Exercise 3

Suppose that you had a circular drum with wave speed c_d and radius a and a violin string with wave speed c_v and length l . In order to make the fundamental frequencies of the drum and the violin the same, how would you choose the length l ?

[TYPO: Replace “had” with “have.”]

Solution

The fundamental frequency of a circular drum is

$$\nu_d = \frac{\alpha_{01} c_d}{a},$$

where $\alpha_{01} \approx 2.40483$ is the first positive zero of J_0 , the zero-order Bessel function of the first kind. The fundamental frequency of a violin string is

$$\nu_v = \frac{\pi c_v}{l},$$

where $\pi \approx 3.14159$ is the first positive zero of sine. Set the two frequencies to be equal and solve for l .

$$\begin{aligned}\nu_v &= \nu_d \\ \frac{\pi c_v}{l} &= \frac{\alpha_{01} c_d}{a} \\ \frac{l}{\pi c_v} &= \frac{a}{\alpha_{01} c_d}\end{aligned}$$

Therefore,

$$l = \frac{\pi a c_v}{\alpha_{01} c_d}.$$