Exercise 1

For each of the following functions, state whether it is even or odd or periodic. If periodic, what is its smallest period?

(a) $\sin ax \quad (a > 0)$

(b) $e^{ax} \quad (a > 0)$

(c) $x^m \quad (m = \text{integer})$

(d) $\tan x^2$

(e) $|\sin(x/b)| \quad (b > 0)$

(f) $x \cos ax \quad (a > 0)$

Solution

Part (a)

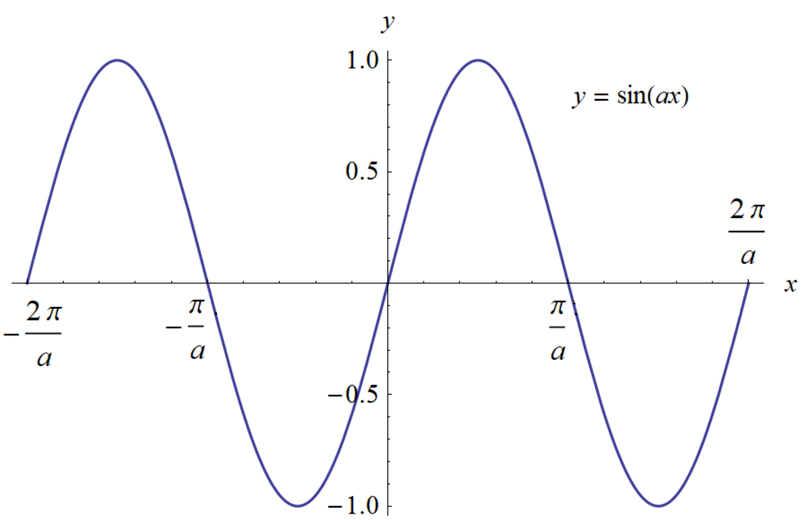

Figure 1: This is a plot of $\sin ax$ vs. x, where $a > 0$.

$y = \sin ax$ is not an even function because it is not symmetric about the y-axis [that is, $y(-x) \neq y(x)$], but it is an odd function because it is symmetric about the origin [that is, $y(-x) = -y(x)$]. It is also a periodic function; the period p is the minimum distance on the x-axis that it takes for the function to repeat itself, so $p = 2\pi/a$.

Part (b)

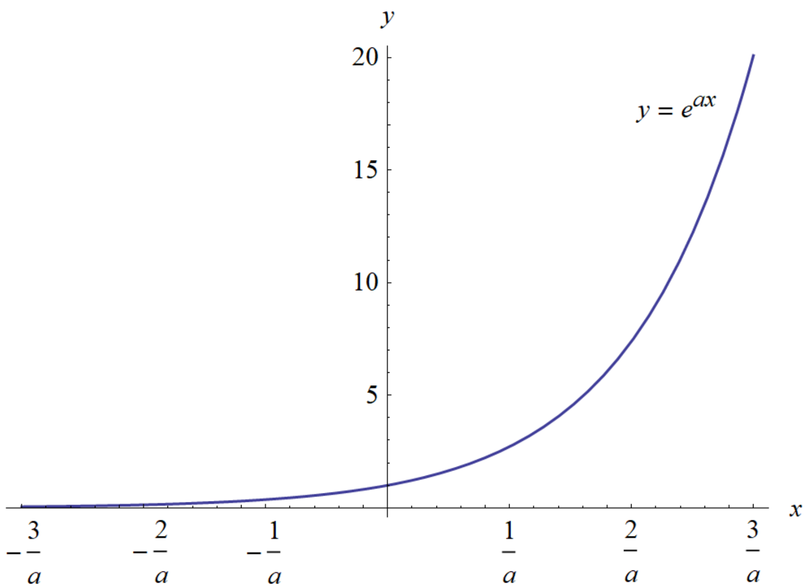

Figure 2: This is a plot of e^{ax} vs. x, where $a > 0$.

$y = e^{ax}$ is not an even function because it is not symmetric about the y-axis [that is, $y(-x) \neq y(x)$], and it is not an odd function because it is not symmetric about the origin [that is, $y(-x) \neq -y(x)$]. e^{ax} is not periodic since it does not repeat itself in any interval of x.

Part (c)

The function $y = x^m$ (m is an integer) behaves differently, depending on whether m is even and positive, even and negative, odd and positive, odd and negative, or zero. Each case will be considered in turn.

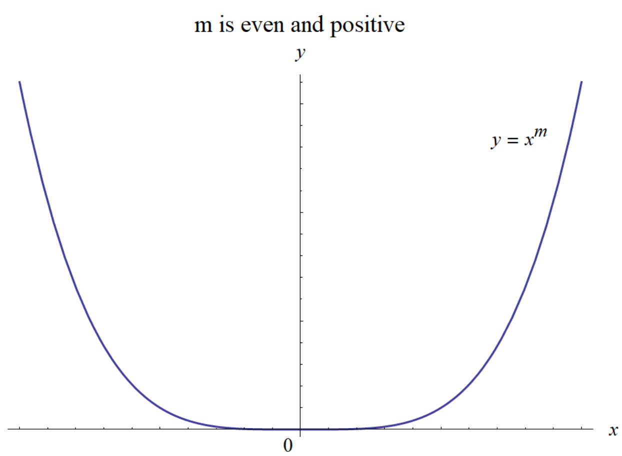

Figure 3: This is the first plot of x^m vs. x.

If m is even and positive, then $y(x) = x^m$ is an even function because it is symmetric about the y-axis. This can be shown algebraically by letting $m = 2k$, where k is a positive integer. Then $y(x) = x^{2k}$ and

$$y(-x) = (-x)^{2k} = [(-x)^2]^k = (x^2)^k = x^{2k} = y(x).$$

It is not an odd function because it is not symmetric about the origin [that is, $y(-x) \neq -y(x)$], and it is not periodic because the function does not repeat itself in any interval of x.

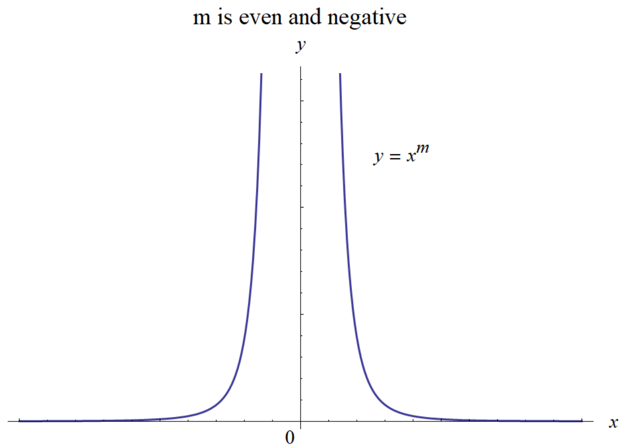

Figure 4: This is the second plot of x^m vs. x.

If m is even and negative, then $y(x) = x^m$ is an even function because it is symmetric about the y-axis. This can be shown algebraically by letting $m = -2k$, where k is a positive integer. Then $y(x) = x^{-2k} = 1/x^{2k}$ and

$$y(-x) = \frac{1}{(-x)^{2k}} = \frac{1}{[(-x)^2]^k} = \frac{1}{(x^2)^k} = \frac{1}{x^{2k}} = y(x).$$

It is not an odd function because it is not symmetric about the origin [that is, $y(-x) \neq -y(x)$], and it is not periodic because the function does not repeat itself in any interval of x.

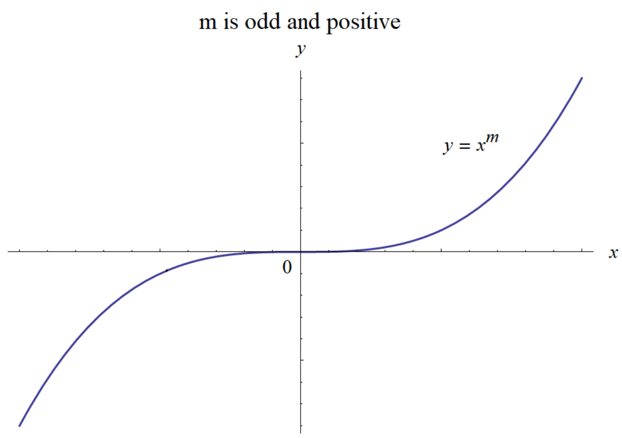

Figure 5: This is the third plot of x^m vs. x.

If m is odd and positive, then $y(x) = x^m$ is not an even function because it is not symmetric about the y-axis [that is, $y(-x) \neq y(x)$]. This can be shown algebraically by letting $m = 2k - 1$, where k is a positive integer. Then $y(x) = x^{2k-1} = x^{2k}/x$ and

$$y(-x) = \frac{(-x)^{2k}}{(-x)} = -\frac{[(-x)^2]^k}{x} = -\frac{(x^2)^k}{x} = -\frac{x^{2k}}{x} = -y(x).$$

It is an odd function because it is symmetric about the origin [that is, $y(-x) = -y(x)$], and it is not periodic because the function does not repeat itself in any interval of x.

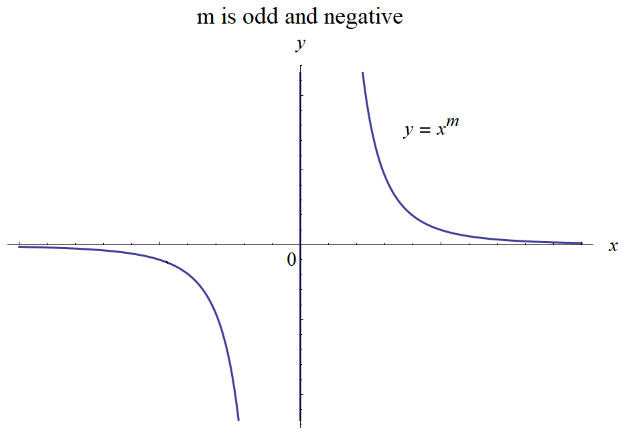

Figure 6: This is the fourth plot of x^m vs. x.

If m is odd and negative, then $y(x) = x^m$ is not an even function because it is not symmetric about the y-axis [that is, $y(-x) \neq y(x)$]. This can be shown algebraically by letting $m = -(2k-1)$, where k is a positive integer. Then $y(x) = x^{-(2k-1)} = x/x^{2k}$ and

$$y(-x) = \frac{(-x)}{(-x)^{2k}} = -\frac{x}{[(-x)^2]^k} = -\frac{x}{(x^2)^k} = -\frac{x}{x^{2k}} = -y(x).$$

It is an odd function because it is symmetric about the origin [that is, $y(-x) = -y(x)$], and it is not periodic because the function does not repeat itself in any interval of x.

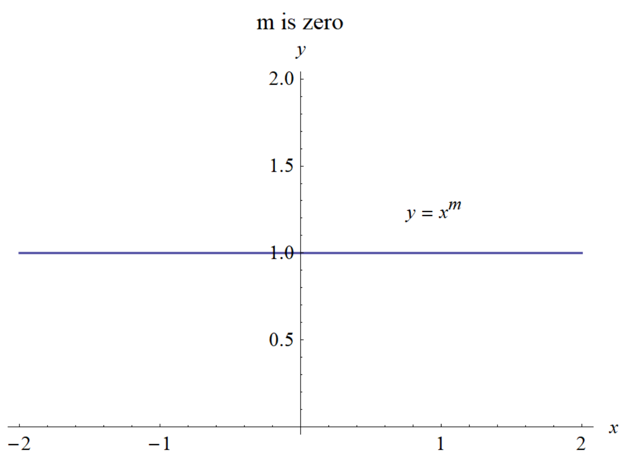

Figure 7: This is the fifth plot of x^m vs. x.

If m is zero, then $y(x) = x^0 = 1$ is an even function because it is symmetric about the y-axis [that is, $y(-x) = y(x)$]. It is not an odd function because it is not symmetric about the origin [that is,

$y(-x) = -y(x)$]. The function is not periodic because there is no smallest interval over which it repeats itself.

To summarize, x^m is an even function if m is even, and it is an odd function if m is odd. x^m is not a periodic function in either case.

Part (d)

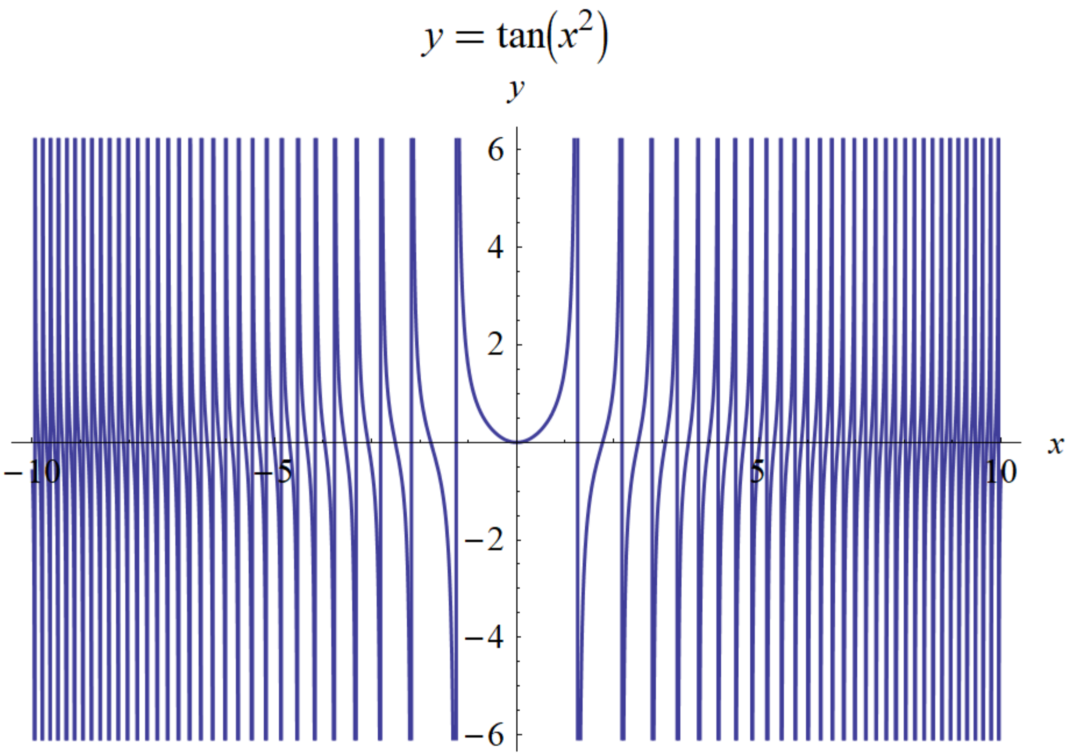

Figure 8: This is a plot of $\tan x^2$ vs. x.

$y = \tan x^2$ is an even function because it is symmetric about the y-axis [that is, $y(-x) = y(x)$], but it is not an odd function because it is not symmetric about the origin [that is, $y(-x) \neq -y(x)$]. The function is not periodic because there is no smallest interval over which it repeats itself.

Part (e)

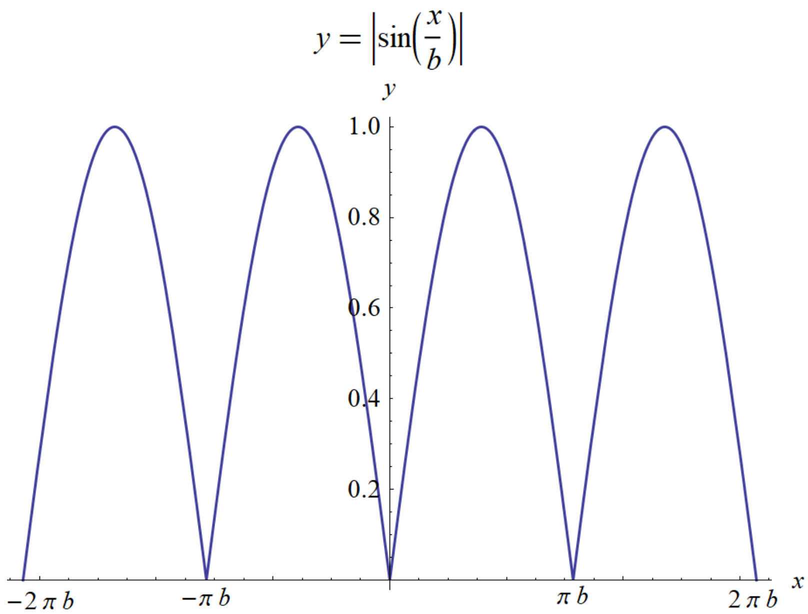

Figure 9: This is a plot of $|\sin(x/b)|$ vs. x, where $b > 0$.

$y = |\sin(x/b)|$ is an even function because it is symmetric about the y-axis [that is, $y(-x) = y(x)$], but it is not an odd function because it is not symmetric about the origin [that is, $y(-x) \neq -y(x)$]. It is a periodic function; the period p is the minimum distance on the x-axis that it takes for the function to repeat itself, so $p = \pi b$.

Part (f)

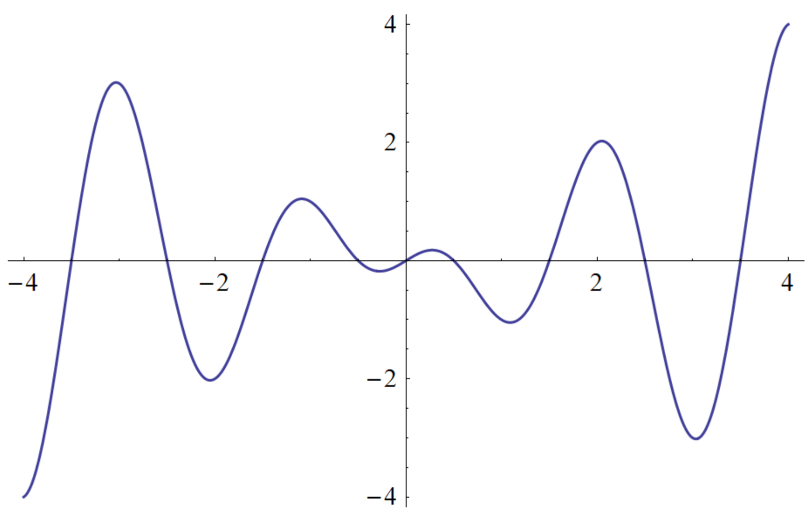

Figure 10: This is a sample plot of $x \cos ax$ vs. x when $a = \pi$.

$y = x \cos ax$ is not an even function because it is not symmetric about the y-axis [that is, $y(-x) \neq y(x)$], but it is an odd function because it is symmetric about the origin [that is, $y(-x) = -y(x)$]. $x \cos ax$ is not periodic since it does not repeat itself in any interval of x.