Exercise 15

Without any computation, predict which of the Fourier coefficients of $|\sin x|$ on the interval $(-\pi, \pi)$ must vanish.

Solution

$f(x) = |\sin x|$ is an even function because

$$f(-x) = |\sin(-x)| = |-\sin x| = |\sin x| = f(x).$$

The Fourier series for $|\sin x|$ on the interval $(-\pi, \pi)$ is

$$|\sin x| = \frac{A_0}{2} + \sum_{n=1}^{\infty}(A_n \cos nx + B_n \sin nx),$$

where the coefficients are given by

$$A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{|\sin x| \cos nx}_{\text{even}}\, dx, \quad n = 0, 1, \ldots$$

$$B_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{|\sin x| \sin nx}_{\text{odd}}\, dx = 0, \quad n = 1, 2, \ldots.$$

Since $|\sin x| \sin nx$ is an odd function and it's being integrated over a symmetric interval, the integral is equal to zero. Therefore, the coefficients of sine will vanish on the interval $(-\pi, \pi)$.