

Exercise 2.1.1

In the next three exercises, interpret $\dot{x} = \sin x$ as a flow on the line.

Find all the fixed points of the flow.

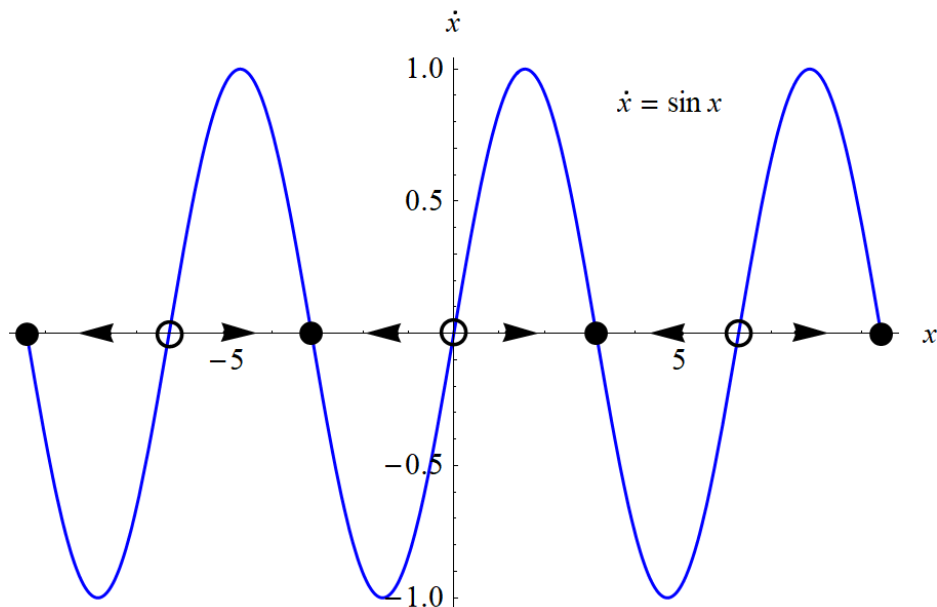
Solution

The fixed points of the flow are the values of x where $\dot{x} = 0$.

$$\sin x^* = 0$$

$$x^* = n\pi, \quad n = 0, \pm 1, \pm 2, \dots$$

In order to determine whether these fixed points are stable or unstable, plot \dot{x} versus x and indicate where the flow is to the right or to the left.



Based on the graph, the fixed points,

$$x^* = 2n\pi, \quad n = 0, \pm 1, \pm 2, \dots,$$

are locally unstable, and the fixed points,

$$x^* = (2n - 1)\pi, \quad n = 0, \pm 1, \pm 2, \dots,$$

are locally stable.