

Exercise 2.2.10

(Fixed points) For each of (a)–(e), find an equation $\dot{x} = f(x)$ with the stated properties, or if there are no examples, explain why not. (In all cases, assume that $f(x)$ is a smooth function.)

- a) Every real number is a fixed point.
- b) Every integer is a fixed point, and there are no others.
- c) There are precisely three fixed points, and all of them are stable.
- d) There are no fixed points.
- e) There are precisely 100 fixed points.

Solution

Fixed points are values of x where $\dot{x} = 0$.

- a) $\dot{x} = 0$
- b) $\dot{x} = \sin \pi x$
- c) There are no examples because there can't be adjacent stable fixed points.
- d) $\dot{x} = 1$
- e) $\dot{x} = \prod_{n=1}^{100} (x - n)$.