

Exercise 2.4.5

Use linear stability analysis to classify the fixed points of the following systems. If linear stability analysis fails because $f'(x^*) = 0$, use a graphical argument to decide the stability.

$$\dot{x} = 1 - e^{-x^2}$$

Solution

The fixed points occur where $\dot{x} = 0$.

$$1 - e^{-x^{*2}} = 0$$

$$-x^{*2} = 0$$

$$x^* = 0$$

Use linear stability analysis to classify this point.

$$f(x) = 1 - e^{-x^2}$$

Differentiate $f(x)$.

$$\begin{aligned} f'(x) &= -e^{-x^2} \cdot \frac{d}{dx}(-x^2) \\ &= 2xe^{-x^2} \end{aligned}$$

As a result,

$$f'(0) = 0 \quad \Rightarrow \quad \text{No conclusion can be made about the stability of } x^* = 0.$$

The graph of \dot{x} versus x below shows that $x^* = 0$ is in fact a half-stable point.

