

Exercise 2.4.6

Use linear stability analysis to classify the fixed points of the following systems. If linear stability analysis fails because $f'(x^*) = 0$, use a graphical argument to decide the stability.

$$\dot{x} = \ln x$$

Solution

The fixed points occur where $\dot{x} = 0$.

$$\ln x^* = 0$$

$$x^* = 1$$

Use linear stability analysis to classify this point.

$$f(x) = \ln x$$

Differentiate $f(x)$.

$$f'(x) = \frac{1}{x}$$

As a result,

$$f'(1) = 1 > 0 \quad \Rightarrow \quad x^* = 1 \text{ is an unstable fixed point.}$$

The graph of \dot{x} versus x below confirms it.

