Exercise 2.7.1

For each of the following vector fields, plot the potential function $V(x)$ and identify all the equilibrium points and their stability.

$$\dot{x} = x(1-x)$$

Solution

The potential function $V(x)$ satisfies

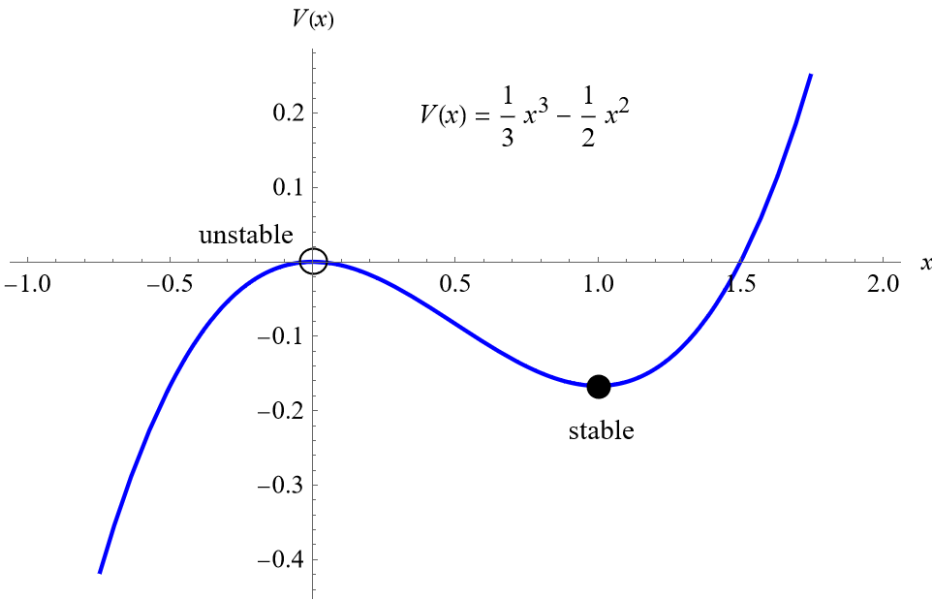
$$\dot{x} = x(1-x) = -\frac{dV}{dx}.$$

Multiply both sides by -1.

$$\frac{dV}{dx} = x^2 - x$$

Integrate both sides with respect to x, setting the integration constant to zero.

$$V(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2$$



The graph of $V(x)$ versus x is to be thought of as a two-dimensional rollercoaster. A particle on the curve at $x^* = 0$ is unstable because the slightest nudge in either direction will send it away from $x^* = 0$ indefinitely. A particle on the curve at $x^* = 1$ that's nudged in either direction will return to $x^* = 1$ because it's stable.