

### Exercise 2.7.3

For each of the following vector fields, plot the potential function  $V(x)$  and identify all the equilibrium points and their stability.

$$\dot{x} = \sin x$$

#### Solution

The potential function  $V(x)$  satisfies

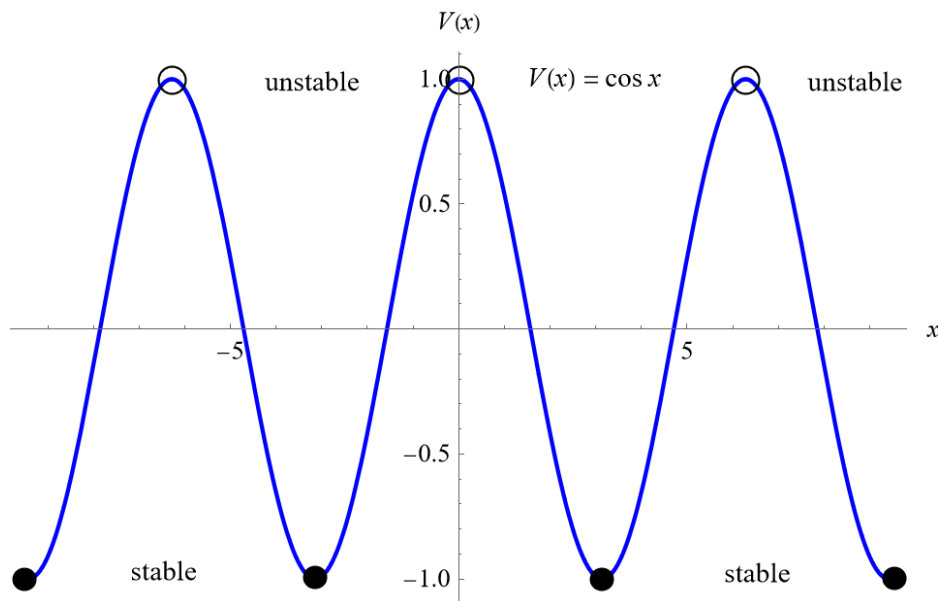
$$\dot{x} = \sin x = -\frac{dV}{dx}.$$

Multiply both sides by  $-1$ .

$$\frac{dV}{dx} = -\sin x$$

Integrate both sides with respect to  $x$ , setting the integration constant to zero.

$$V(x) = \cos x$$



The graph of  $V(x)$  versus  $x$  is to be thought of as a two-dimensional rollercoaster. A particle on the curve at  $x^* = 2n\pi$ , where  $n = 0, \pm 1, \pm 2, \dots$ , is unstable because the slightest nudge in either direction will send it away from  $x^* = 2n\pi$  indefinitely. A particle on the curve at  $x^* = (2n + 1)\pi$  that's nudged in either direction will return to  $x^* = (2n + 1)\pi$  because it's stable.