Exercise 2.7.4

For each of the following vector fields, plot the potential function $V(x)$ and identify all the equilibrium points and their stability.

$$\dot{x} = 2 + \sin x$$

Solution

The potential function $V(x)$ satisfies

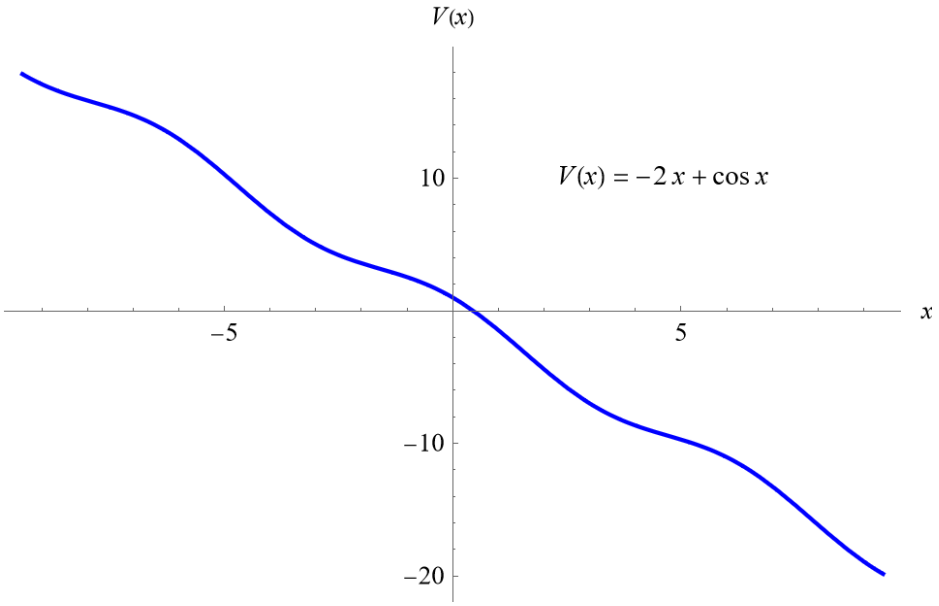
$$\dot{x} = 2 + \sin x = -\frac{dV}{dx}.$$

Multiply both sides by -1.

$$\frac{dV}{dx} = -2 - \sin x$$

Integrate both sides with respect to x, setting the integration constant to zero.

$$V(x) = -2x + \cos x$$



The graph of $V(x)$ versus x is to be thought of as a two-dimensional rollercoaster. A particle placed anywhere on the curve will roll indefinitely to the right because the slope is never zero; in other words, there are no equilibrium points.