

## Exercise 2.7.5

For each of the following vector fields, plot the potential function  $V(x)$  and identify all the equilibrium points and their stability.

$$\dot{x} = -\sinh x$$

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### Solution

The potential function  $V(x)$  satisfies

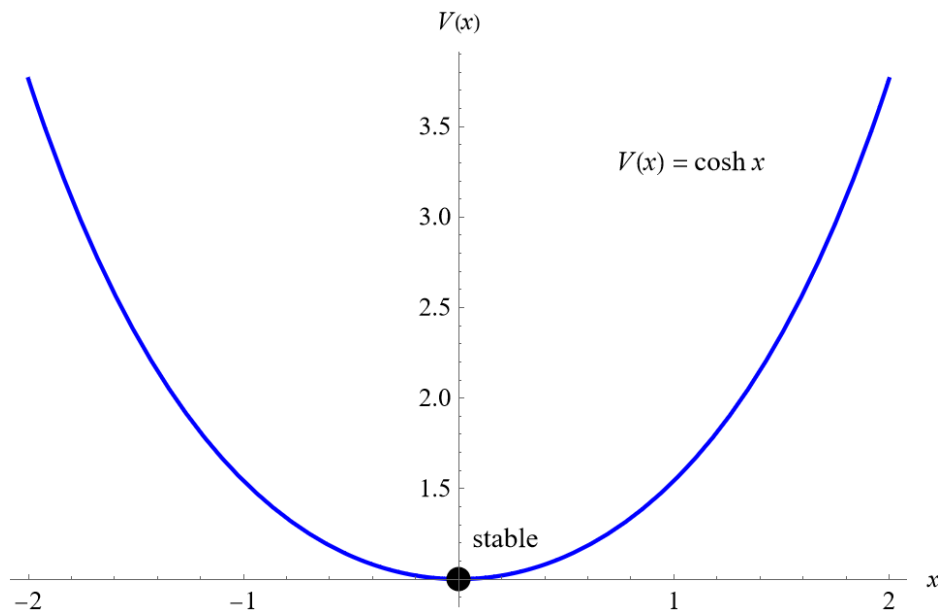
$$\dot{x} = -\sinh x = -\frac{dV}{dx}.$$

Multiply both sides by  $-1$ .

$$\frac{dV}{dx} = \sinh x$$

Integrate both sides with respect to  $x$ , setting the integration constant to zero.

$$V(x) = \cosh x$$



The graph of  $V(x)$  versus  $x$  is to be thought of as a two-dimensional rollercoaster. A particle on the curve at  $x^* = 0$  that's nudged in either direction will return to  $x^* = 0$  because it's stable.