

Problem 1.1

Given the two vectors $\mathbf{b} = \hat{\mathbf{x}} + \hat{\mathbf{y}}$ and $\mathbf{c} = \hat{\mathbf{x}} + \hat{\mathbf{z}}$ find $\mathbf{b} + \mathbf{c}$, $5\mathbf{b} + 2\mathbf{c}$, $\mathbf{b} \cdot \mathbf{c}$, and $\mathbf{b} \times \mathbf{c}$.

Solution

Write the given vectors as follows.

$$\mathbf{b} = \langle 1, 1, 0 \rangle$$

$$\mathbf{c} = \langle 1, 0, 1 \rangle$$

Now make the calculations.

$$\begin{aligned}\mathbf{b} + \mathbf{c} &= \langle 1, 1, 0 \rangle + \langle 1, 0, 1 \rangle \\ &= \langle 1 + 1, 1 + 0, 0 + 1 \rangle \\ &= \langle 2, 1, 1 \rangle\end{aligned}$$

$$\begin{aligned}5\mathbf{b} + 2\mathbf{c} &= 5\langle 1, 1, 0 \rangle + 2\langle 1, 0, 1 \rangle \\ &= \langle 5, 5, 0 \rangle + \langle 2, 0, 2 \rangle \\ &= \langle 5 + 2, 5 + 0, 0 + 2 \rangle \\ &= \langle 7, 5, 2 \rangle\end{aligned}$$

$$\begin{aligned}\mathbf{b} \cdot \mathbf{c} &= \langle 1, 1, 0 \rangle \cdot \langle 1, 0, 1 \rangle \\ &= (1)(1) + (1)(0) + (0)(1) \\ &= 1\end{aligned}$$

$$\begin{aligned}\mathbf{b} \times \mathbf{c} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \\ &= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \hat{\mathbf{x}} - \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \hat{\mathbf{y}} + \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \hat{\mathbf{z}} \\ &= [(1)(1) - (0)(0)]\hat{\mathbf{x}} - [(1)(1) - (0)(1)]\hat{\mathbf{y}} + [(1)(0) - (1)(1)]\hat{\mathbf{z}} \\ &= \hat{\mathbf{x}} - \hat{\mathbf{y}} - \hat{\mathbf{z}} \\ &= \langle 1, -1, -1 \rangle\end{aligned}$$