

Problem 1.4

One of the many uses of the scalar product is to find the angle between two given vectors. Find the angle between the vectors $\mathbf{b} = (1, 2, 4)$ and $\mathbf{c} = (4, 2, 1)$ by evaluating their scalar product.

Solution

The scalar product (also known as the dot product) of two vectors, \mathbf{b} and \mathbf{c} , is related to the angle θ between them by

$$\mathbf{b} \cdot \mathbf{c} = |\mathbf{b}||\mathbf{c}| \cos \theta.$$

Solve for θ .

$$\begin{aligned} \cos \theta &= \frac{\mathbf{b} \cdot \mathbf{c}}{|\mathbf{b}||\mathbf{c}|} \\ \theta &= \cos^{-1} \left(\frac{\mathbf{b} \cdot \mathbf{c}}{|\mathbf{b}||\mathbf{c}|} \right) \end{aligned}$$

Now that the formula is known, plug in the numbers.

$$\begin{aligned} \theta &= \cos^{-1} \left(\frac{\langle 1, 2, 4 \rangle \cdot \langle 4, 2, 1 \rangle}{\sqrt{1^2 + 2^2 + 4^2} \sqrt{4^2 + 2^2 + 1^2}} \right) \\ &= \cos^{-1} \left(\frac{1 \cdot 4 + 2 \cdot 2 + 4 \cdot 1}{\sqrt{21} \sqrt{21}} \right) \\ &= \cos^{-1} \left(\frac{12}{21} \right) \\ &\approx 52.5^\circ \end{aligned}$$