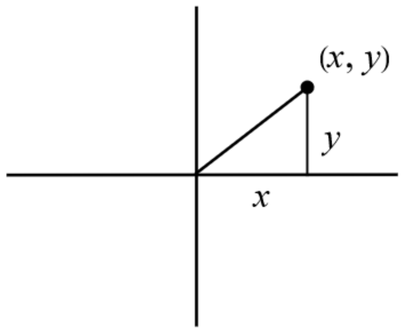
# Problem 1.42

Prove that the transformations from rectangular to polar coordinates and vice versa are given by the four equations (1.37). Explain why the equation for $\phi$ is not quite complete and give a complete version.

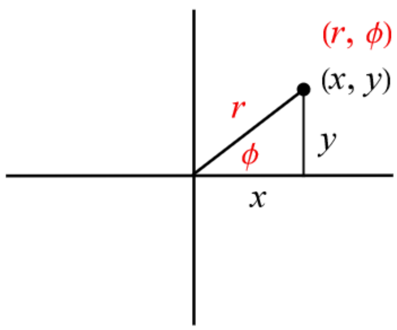
---

**Solution**

Cartesian coordinates $(x, y)$ are used to represent the location of a point. $x$ represents the horizontal distance from the origin, and $y$ represents the vertical distance from the origin.




Alternatively, polar coordinates $(r, \phi)$ can be used to represent the location of a point.




$r$ is related to $x$ and $y$ by the Pythagorean theorem.

$$r^2 = x^2 + y^2$$

Taking the square root of both sides gives

$$r = \pm\sqrt{x^2 + y^2}.$$

Only the positive root is needed; the negative root is redundant.

$$r = \sqrt{x^2 + y^2}$$

Based on the definitions of the trigonometric functions,

$$\cos\phi = \frac{x}{r} \quad \rightarrow \quad x = r\cos\phi$$

$$\sin\phi = \frac{y}{r} \quad \rightarrow \quad y = r\sin\phi$$

$$\tan\phi = \frac{y}{x} \quad \Rightarrow \quad \phi = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x \text{ and } y \text{ are positive (Quadrant I)} \\ \pi + \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x \text{ is negative and } y \text{ is positive (Quadrant II)} \\ \pi + \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x \text{ and } y \text{ are negative (Quadrant III)} \\ \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x \text{ is positive and } y \text{ is negative (Quadrant IV)} \end{cases}.$$

These conditions for the last equation are due to the fact that the inverse tangent only yields a value for $\phi$ between $-\pi/2$ and $\pi/2$. This is fine if the point lies in the first or fourth quadrants. If the point lies in the second or third quadrants, though, then $\pi$ has to be added to compensate for this limited range.