Problem 1.6

By evaluating their dot product, find the values of the scalar s for which the two vectors $\mathbf{b} = \hat{\mathbf{x}} + s\hat{\mathbf{y}}$ and $\mathbf{c} = \hat{\mathbf{x}} - s\hat{\mathbf{y}}$ are orthogonal. (Remember that two vectors are orthogonal if and only if their dot product is zero.) Explain your answers with a sketch.

Solution

Take the dot product of $\mathbf{b} = \langle 1, s\rangle$ and $\mathbf{c} = \langle 1, -s\rangle$.

$$\begin{aligned}\mathbf{b}\cdot\mathbf{c} &= \langle 1, s\rangle \cdot \langle 1, -s\rangle \\ &= (1)(1) + (s)(-s) \\ &= 1 - s^2\end{aligned}$$

If $\mathbf{b}$ and $\mathbf{c}$ are orthogonal, then their dot product must be zero.

$$\mathbf{b}\cdot\mathbf{c} = 0$$

$$1 - s^2 = 0$$

$$s = \{\pm 1\}$$