

## Problem 1.24

Assume a pulse occurs at integer times and lasts for 1 second. It has a random amplitude with the probability of having the amplitude equal 1 or  $-1$  being  $p(1) = p(-1) = 1/2$ . What is the mean value and the mean square value of the amplitude?

### Solution

The mean value of a wave  $x(t)$  is defined to be

$$\bar{x} = \frac{\int x dt}{\int dt},$$

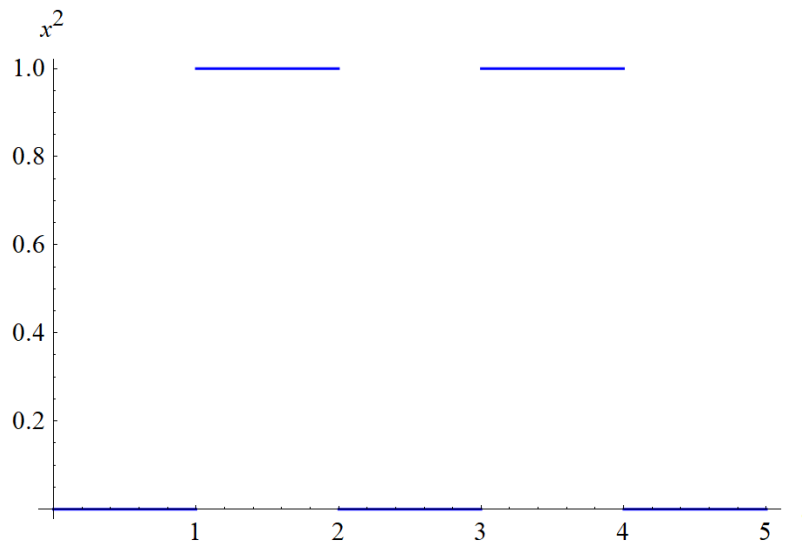
and the mean square value is defined as

$$\overline{x^2} = \frac{\int x^2 dt}{\int dt}.$$

On average,  $x$  is  $+1$  the same amount of time as it is  $-1$ , so the mean value of the wave is zero.

$$\bar{x} = 0$$

On the other hand,  $(1)^2 = 1$  and  $(-1)^2 = 1$ , so  $x^2$  is a series of positive rectangular pulses with amplitude 1 and period 2.



The mean square value for the whole wave can be found by integrating over one cycle.

$$\begin{aligned} \overline{x^2} &= \frac{\int_0^1 (0)^2 dt + \int_1^2 (\pm 1)^2 dt}{\int_0^2 dt} \\ &= \frac{1}{2} \end{aligned}$$