

## Exercise 1

Find the closed form function for the following Taylor series:

$$f(x) = 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \dots$$

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### Solution

$$\begin{aligned} f(x) &= 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \dots \\ f(x) &= \frac{(2x)^1}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \dots \\ f(x) &= \sum_{n=1}^{\infty} \frac{(2x)^n}{n!} \\ f(x) &= \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} - 1 \end{aligned}$$

Therefore,

$$f(x) = e^{2x} - 1.$$