

Exercise 5

Find the closed form function for the following Taylor series:

$$f(x) = 3x - \frac{9}{2}x^3 + \frac{81}{40}x^5 - \frac{243}{560}x^7 + \dots$$

Solution

$$\begin{aligned} f(x) &= 3x - \frac{9}{2}x^3 + \frac{81}{40}x^5 - \frac{243}{560}x^7 + \dots \\ f(x) &= \frac{(3x)^1}{1!} - \frac{(3x)^3}{3!}x^3 + \frac{(3x)^5}{5!}x^5 - \frac{(3x)^7}{7!}x^7 + \dots \\ f(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (3x)^{2n+1} \end{aligned}$$

Therefore,

$$f(x) = \sin 3x.$$