

Exercise 8

Find the closed form function for the following Taylor series:

$$f(x) = \frac{9}{2}x^2 + \frac{27}{8}x^4 + \frac{81}{80}x^6 + \dots$$

Solution

$$f(x) = \frac{9}{2}x^2 + \frac{27}{8}x^4 + \frac{81}{80}x^6 + \dots$$

$$f(x) = \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} + \frac{(3x)^6}{6!} + \dots$$

$$f(x) = -1 + \frac{(3x)^0}{0!} + \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} + \frac{(3x)^6}{6!} + \dots$$

$$f(x) = -1 + \sum_{n=0}^{\infty} \frac{1}{(2n)!} (3x)^{2n}$$

Therefore,

$$f(x) = \cosh 3x - 1.$$